Integration Bee 2023

Mathematics Student Society with IMC

2023

Rules

- There will be a 3 minute time limit for each question (unless the judges say otherwise).
- For indefinite integrals (integrals without bounds), you must write $+C$ or mathematically equivalent.
- Your answer will be considered submitted once a box or circle has been drawn around it. No changes can be made after this point.
- If an incorrect answer is given, the contestant will not be able to write nor submit another answer for 30 seconds.
- An answer will be marked correct iff it matches the known solution almost everywhere on a reasonable domain.

Basic

Question 0

$$\int x^{42} + 2x + 1 \, dx$$

Question 0

$$\int x^{42} + 2x + 1\, dx = \frac{x^{43}}{43} + x^2 + x + C$$

Question 1

$$\int \left(\frac{x}{2023} + 1\right)^{2023} dx$$

Question 1

$$\int \left(\frac{x}{2023}+1\right)^{2023} dx = \frac{2023}{2024}\left(\frac{x}{2023}+1\right)^{2024}+C$$

Question 2

$$\int 5x^2 + \sin(2x) + 3\, dx$$

Question 2

$$\int 5x^2 + \sin(2x) + 3\,dx = \frac{5x^3}{3} - \frac{\cos(2x)}{2} + 3x + C$$

Question 3

$$\int (e^x)^2 \; dx$$

Question 3

$$\int (e^x)^2 \, dx = \frac{e^{2x}}{2} + C = \frac{(e^x)^2}{2} + C$$

Question 4

$$\int \left(x - \sqrt[3]{2}\right)\left(x^2 + \sqrt[3]{2}x + 2^{\frac{2}{3}}\right)\, dx$$

Question 4

$$\int \left(x - \sqrt[3]{2}\right)\left(x^2 + \sqrt[3]{2}x + 2^{\frac{2}{3}}\right)\,dx = \frac{x^4}{4} - 2x + C$$

Question 5

$$\int 9x \sin\left(2x^2\right)\, dx$$

Question 5

$$\int 9x \sin\left(2x^2\right)\, dx = -\frac{9}{4}\cos\left(2x^2\right) + C$$

Question 6

$$\int_0^1 \frac{8}{3}x^4 + \frac{4}{5}x^3 + \frac{7}{3}x^2 - \frac{3}{5}x - \frac{11}{180}\,dx$$

Question 6

$$\int_0^1 \frac{8}{3}x^4 + \frac{4}{5}x^3 + \frac{7}{3}x^2 - \frac{3}{5}x - \frac{11}{180}\,dx = \frac{23}{20}$$

Question 7

$$\int (x+1)^3 + (x-1)^{-3}\, dx$$

Question 7

$$\int (x+1)^3 + (x-1)^{-3}\,dx = \frac{(x+1)^4}{4} - \frac{(x-1)^{-2}}{2} + C$$

Question 8

$$\int -7x^2 \left(x^3 + 7\right)^7 \, dx$$

Question 8

$$\int -7x^2\left(x^3+7\right)^7\,dx = -\frac{7}{24}\left(x^3+7\right)^8 + C$$

Question 9

$$\int_0^{2\ln 2} \sqrt{e^x}\, dx$$

Question 9

$$\int_0^{2\ln 2} \sqrt{e^x}\, dx = 2$$

Question 10

$$\int_0^1 \cos(\pi x) + \sin(\pi x) + \cos(\pi x)\sin(\pi x)\, dx$$

Question 10

$$\int_0^1 \cos(\pi x) + \sin(\pi x) + \cos(\pi x)\sin(\pi x)\, dx = \frac{2}{\pi}$$

Question 11

$$\int \cos^2 x - \sin^2 x \, dx$$

Question 11

$$\int \cos^2 x - \sin^2 x \, dx = \cos x \sin x + C = \frac{\sin(2x)}{2} + C$$

Question 12

$$\int_{-\frac{1}{2}\ln(e^2)}^{1} \operatorname{sign}(x)\cosh x \, dx$$

Question 12

$$\int_{-\frac{1}{2}\ln(e^2)}^{1} \operatorname{sign}(x)\cosh x \, dx = 0$$

Question 13

$$\int 2023^x + x^{2023}\,dx$$

Question 13

$$\int 2023^x + x^{2023}\,dx = \frac{x^{2024}}{2024} + \frac{2023^x}{\ln 2023} + C$$

Intermediate

Question 14

$$\int \left(2x^2 + 1\right) \cos x \, dx$$

Question 14

$$\int (2x^2 + 1) \cos x \, dx = (2x^3 - 3) \sin x + 4x \cos x + C$$

Question 15

$$\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} x^2 + 3xy^2 + \sin y \, dx \, dy$$

Question 15

$$\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} x^2 + 3xy^2 + \sin y \, dx \, dy = \frac{4\pi^4}{3}$$

Question 16

$$\int \ln(x-1)\,dx$$

Question 16

$$\int \ln(x-1)\,dx = (x-1)\ln(x-1) - x + C$$

Question 17

$$\int e^x \sqrt{1+e^x}\, dx$$

Question 17

$$\int e^x \sqrt{1 + e^x}\, dx = \frac{2}{3}(1 + e^x)^{\frac{3}{2}} + C$$

Question 18

$$\int \frac{d\int}{\sqrt{1+\int}}$$

Question 18

$$\int \frac{d\int}{\sqrt{1+\int}} = 2\sqrt{\int + 1} + x$$

Question 19

$$\iint_{x^2+y^2\leq\pi^2} 4x^2 + 8xy + 4y^2 \, dx \, dy$$

Question 19

$$\iint_{x^2+y^2\leq\pi^2} 4x^2 + 8xy + 4y^2\,dx\,dy = 2\pi^5$$

Question 20

$$\int e^x \sin(\pi x)\, dx$$

Question 20

$$\int e^x \sin(\pi x)\, dx = \frac{e^x(\sin(\pi x) - \pi \cos(\pi x))}{1+\pi^2} + C$$

Question 21

$$\int \frac{1-x}{1-\sqrt{x}}\, dx$$

Question 21

$$\int \frac{1-x}{1-\sqrt{x}}\,dx = \frac{2x^{\frac{3}{2}}}{3} + x + C$$

Question 22

$$\int \tan^2 x \, dx$$

Question 22

$$\int \tan^2 x \, dx = \tan x - x + C$$

Question 23

$$\int_0^1 x^3 e^{\left(x^2\right)}\, dx$$

Question 23

$$\int_0^1 x^3 e^{\left(x^2\right)}\, dx = \frac{1}{2}$$

Question 24

$$\int \frac{1}{1+e^x}\,dx$$

Question 24

$$\int \frac{1}{1+e^x}\,dx = x - \ln(1+e^x) + C$$

Question 25

$$\int \frac{1}{x \ln x} + \frac{\ln x}{x} \, dx$$

Question 25

$$\int \frac{1}{x \ln x} + \frac{\ln x}{x}\, dx = \frac{(\ln x)^2}{x} + \ln(\ln x) + C$$

Question 26

$$\int \left(e^{\sqrt{x}}\right)^2 \, dx$$

Question 26

$$\int \left(e^{\sqrt{x}}\right)^2 \, dx = \frac{\left(2\sqrt{x}-1\right) e^{2\sqrt{x}}}{2} + C$$

Question 27

$$\int x\sqrt{x-1}\,dx$$

Question 27

$$\int x\sqrt{x-1}\,dx = \frac{2x(x-1)^{\frac{3}{2}}}{5} + \frac{4(x-1)^{\frac{3}{2}}}{15} + C$$

Question 28

$$\int (\ln x + 1)\, x^{2x}\, dx$$

Question 28

$$\int (\ln x + 1)\, x^{2x}\, dx = \frac{x^{2x}}{2} + C$$

Question 29

$$\int_{-\infty}^{\infty} x \cos\left(x^2\right) e^{-x^2}\, dx$$

Question 29

$$\int_{-\infty}^{\infty} x \cos\left(x^2\right) e^{-x^2}\, dx = 0$$

Difficult

Question 30

$$\int_0^{\infty} x \cos\left(x^2\right) e^{-x^2}\, dx$$

Question 30

$$\int_0^\infty x \cos\left(x^2\right) e^{-x^2}\, dx = \frac{1}{4}$$

Question 31

$$\int_0^3 \left(9x^2 + 3\right) \text{sign}(\sin(\pi x))\, dx$$

Question 31

$$\int_0^3 (9x^2+3)\,\mathrm{sign}(\sin(\pi x))\,dx = 42$$

Question 32

$$\int_0^{\frac{\pi}{2}} \frac{8\sin x}{\sin x + \cos x}\, dx$$

Question 32

$$\int_0^{\frac{\pi}{2}} \frac{8\sin x}{\sin x + \cos x}\,dx = 2\pi$$

Question 33

$$\int \sin \sqrt{x} \, dx$$

Question 33

$$\int \sin\sqrt{x}\,dx = 2\sin\sqrt{x} - 2\sqrt{x}\cos\sqrt{x}$$

Question 34

$$\int_0^\infty \frac{x}{e^x - 1}\, dx$$

Question 34

$$\int_0^\infty \frac{x}{e^x - 1}\,dx = \frac{\pi^2}{6}$$